A quasi-topological characterization and study of holomorphic dynamical systems

A holomorphic dynamical system could be

1] a foliation of real codimension two with a covering by product boxes where the leaves are horizontal , the verticals are riemann surfaces and the path holonomy mappings are holomorphic homeomorphisms.
2] generated by a countable set of holomorphic homeomorphisms [and their inverses] each defined on an open set of a riemann surface, mapping it back into itself . These are composed whenever composition is defined. to generate a holomorphic equivalence relation.
3] any countable collection of partially defined holomorphic maps of a riemann surface to itself .eg a single rational mapping of the riemann sphere.

Note an example of type1] determines an example of type 2]. An example of type 3] determines one of type 2] by restricting each map or its inverse branches to various univalent open sets.

Sometimes one wants to classify holomorphic dynamical systems as they are presented. Other times just in terms of classifying the holomorphic equivalence relation generated. One also wants to know if there are deformations among holomorphic dynamical systems preserving the topological structure of the equivalence classes.

The smooth measure class is preserved by all of the maps in a holomorphic dynamical system. Therefore one can divide the underlying riemann surface [mod sets of measure zero] into the "transient part" where there is a measurable cross section of the equivalence relation and its complement which is called the "recurrent part".

The Ahlfors-Bers-Morrey measurable Riemann mapping theorem, which I learned from the group around Lippa in the late 70's, allows one to give a topological characterization of holomorphic dynamical systems:they are those which against some quasi-conformal background structure only have bounded conformal distortion The measurable Riemann mapping theorem also allows one to say something about their quasi-conformal deformations : essentially there are always non trivial deformations if the recurrent part and the transient part each have positive measure. Sometimes there is a converse, namely the existence of a deformation implies there is a transient part e.g. Kleinian groups. Studying rational mappings leads one to speculate that the converse may hold for general holomorphic dynamical systems unless the deformation is supported in a linear model ; eg the linear expanding maps of torii factored down to the Riemann sphere.